A Recipe for Orbit Propagation: Perturbed Keplerian Orbit

This algorithm only computes the lowest-order (J_2) secular perturbations and applies to a low-earth orbiting satellite. If you are interested in the non-secular perturbations it is better to integrate Newton's equations of motion. Luni-solar forces also effect all of the elements – see the GTARG paper if you are interested in an algorithm that includes these forces.

Input: Cartesian state vector (x, y, z, v_x, v_y, v_z); start time t_{start}; end time t_{end}; time step Δt; satellite mass m; cross sectional area A; Atmospheric density ρ (in general it is more common to calculate this but the formulas are very complicated).

Define constants: $\mu = 398600.436\ km^3 / \sec^2$; $C_D = 2.2$ (Coefficient of Drag); $\omega_e = 2\pi / 86164\ radians / second$ (Earth spin rate); $J_2 = 0.00108263$

Convert state vector to Keplerian Elements $(a, e, i, \Omega, \omega, M)$

Let $t = t_{start}$

While $t < t_{end}$, Repeat the following

$$n = \sqrt{\mu / a^3}$$

Compute the secular perturbations:

$$\dot{a} = -\frac{\rho A C_D \sqrt{\mu a}}{m}\left[1 - (\omega_e / n)\cos i\right]^2 \quad \text{(drag perturbation)}$$

$\dot{a}_{secular} = 0$ (there is no J_2 perturbation on semi-major axis)

$\dot{e}_{secular} = 0$ (there is no J_2 perturbation on eccentricity)

$\dot{i}_{secular} = 0$ (there is no J_2 perturbation on the inclination)

$$\dot{\Omega}_{secular} = -\frac{3\mu J_2 n \cos i}{2(1 - e^2)^2}$$

$$\dot{\omega}_{secular} = \frac{3\mu J_2 n (5\cos^2 i - 1)}{4(1 - e^2)^2} = \frac{3J_2 \mu n}{4(1 - e^2)^2}(4 - 5\sin^2 i)$$ [*]

$$\dot{M}_{secular} = n\left(1 - \frac{3\mu J_2 (1 - 3\sin^2 i \sin^2 \omega)}{2(1 - e)^3}\right)$$

Update the orbital elements:

$$a \to a + \dot{a}_{drag}\Delta t + \dot{a}_{secular}\Delta t$$

$$e \to e + \dot{e}_{secular}\Delta t$$

$$i \to i + \dot{i}_{secular}\Delta t$$

$$\Omega \to \Omega + \dot{\Omega}_{secular}\Delta t$$

$$\omega \to \omega + \dot{\omega}_{secular}\Delta t$$

$$M \to M + \dot{M}_{secular}\Delta t$$

Update the time:

$$t \to t + \Delta t$$

If desired, calculate the ground track longitude and latitude

If desired, print out the time, and the orbital elements

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[*] These two expressions are equivalent because of trigonometric identities.