Math 103 Practice Problems for the Final

May 21, 2008

These problems are a sample of the kinds of problems that may appear on the final exam. Some answers are included to indicate what is expected. Problems that require a summary statement are marked with Sum . The summary statements should be written in complete sentences and they should include the units of measurement for all quantities mentioned in the summary.

This version divides the problems according to the sections of the book.

1 Section 1.1: Linear Equations and Inequalities

1.1 `[Done in 103L, no webwork version]` RoBoCo Costume Inc. plans to launch a major campaign to sell Robby the Robot costumes for Halloween. The price-demand equation for Robby costumes is

$$d = 120 - p.$$

The demand d is the number of Robby costumes (in thousands) that can be sold at a price of p dollars.

(a) Sum How many costumes can be sold at a price of $80.00?

(b) Sum What price should be charged if the demand is 100,000 Robby costumes?

(c) Sum If the price increases by $1.00, by how much does the demand decrease?

1.2 `[On Webwork]` The price-demand equation for gasoline is

$$0.2x + 5p = 80,$$

where p is the price per gallon and x is the daily demand measured in millions of gallons.

a. Write the demand $f(p)$ as a function of price.

$f(p) =$

b. Sum What is the demand if the price is \$4.00 per gallon? Use the correct units to express your answer.

1.3 `[On Webwork]` The price-demand equation for gasoline is

$$0.1x + 5p = 40,$$

where p is the price per gallon and x is the daily demand measured in millions of gallons.

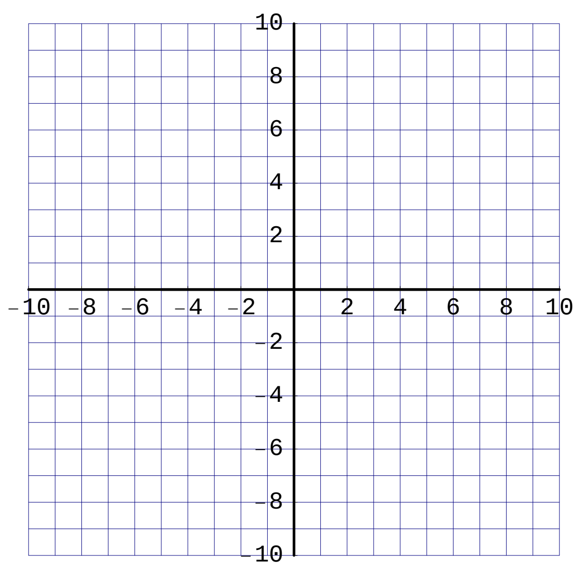
a. Write the demand $f(p)$ as a function of price.

$f(p) =$

b. Sum What is the demand if the price is \$4.00 per gallon? Use the correct units to express your answer.

2 Section 1.2: Graphs and Lines

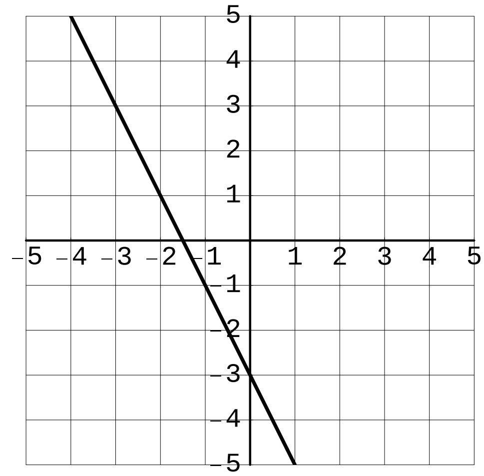
2.1 `[Done in 103L, variant on webwork.]` Plot the points $(-4, 6)$, $(1, 3)$ on the graph below and make an **accurate** drawing of the straight line passing through the two points.



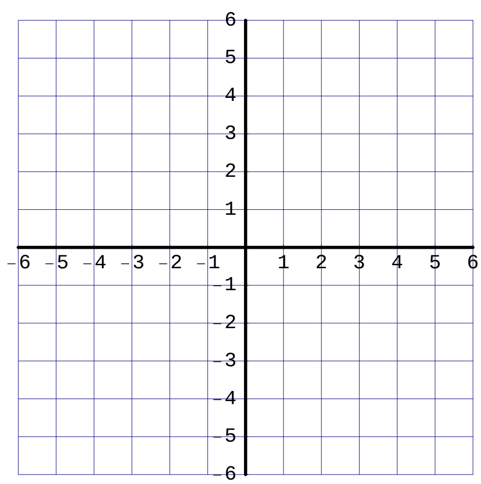
Find the slope of the line:	
What is the y-intercept?	

2.2 `[Done in 103L, variant on webwork.]` The graph of a linear function $f(x)$ is shown below.

Find the slope of the line:	
Find $f(-2)$	
What is the y-intercept?	



2.3 `[Done in 103L, variant on webwork.]` Draw an accurate graph of the function $f(x) = -\frac{1}{2}x + 1$. Your graph should clearly show the intercepts, and the point $(-4, f(-4))$.



3 Section 2.1: Functions

3.1 `[On Webwork]` Let $f(x) = \dfrac{3x-1}{2x+4}$

(a) Find $f(2)$.

(b) Write the domain of the function in interval notation.

(c) Prove that $y = \frac{3}{2}$ is not in the range of $f(x)$.

3.2 `[On Webwork]` The range of the rational function $f(x)$ includes all but one number. What is that number? Prove that the number is not in the range of $f(x)$.

$$f(x) = \frac{2x-1}{5x+1}.$$

3.3 `[On Webwork]` Let

$$f(x) = x^2 - 3x.$$

Evaluate and simplify

$$f(a+1) - f(a).$$

3.4 `[On Webwork]` Let $f(x) = 2x^2$. Evaluate and simplify the expression

$$f(3+h) - f(3).$$

3.5 `[On Webwork]` Let $f(x) = x^2$. Evaluate and simplify the expression

$$\frac{f(-2+h) - f(-2)}{h}.$$

3.6 `[On Webwork]` Let $f(x) = 2x + 3$. Evaluate and simplify the expression

$$f(-1+h) - f(-1).$$

3.7 `[Done in 103L, variant on webwork.]` Write the domain of the function $f(x) = \sqrt{x-5}$ in interval notation.

3.8 `[Done in 103L, no webwork version]` The cost to produce x bookends is

$$C(x) = 350 + 3x,$$

where $C(x)$ is given in dollars.

- $\boxed{\text{Sum}}$ Evaluate and interpret $C(10)$.
- $\boxed{\text{Sum}}$ Write a summary for the statement $C(40) = 470$.

- Sum What is the cost of producing the 11th bookend?

3.9 `[On Webwork]` The cost to produce x doorstops is

$$C(x) = 120 + .50x,$$

where $C(x)$ is given in dollars.

- Evaluate and simplify $C(x+1) - C(x)$.
- Sum What is the cost of producing the 45th door stop?

3.10 `[Done in 103L, no webwork version]` A music company sells CDs for a particular artist. The company has advertising costs of $4000 and recording costs of $10,000. Their cost for manufacturing, royalties, and distribution are $5.50 per CD. They sell the CDs to Maga-Mart for $7.20 each.

(a) Sum What are the fixed costs?

(b) Sum What are the variable costs?

(c) What is the equation for the cost function for x CDs?
$C(x) =$

(d) What is the equation for the revenue function?
$R(x) =$

(e) Sum How many CDs must the company sell to break even?

3.11 `[On Webwork]` Great Neck Pencil Inc. manufactures wooden pencils. The fixed costs for setting up the wood lathes, drills, and yellow paint machine are $535.00. The variable cost is $0.12 per pencil.

a. Write an expression for the cost function $C(x)$, where x is the number pencils manufactured.

$C(x) =$

b. Sum What is the total cost to manufacture 2,000 wooden pencils?

3.12 `[On Webwork]` The price-demand equation for avocados is

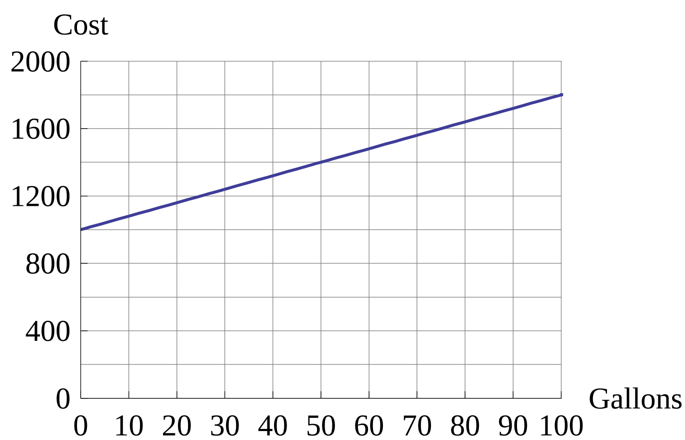
$$20p + x = 25,$$

where p is the price of an avocado and x is the weekly demand (in thousands) for avocados.

Write an expression for revenue as a function of the weekly **demand** for avocados.

$R(x) =$

3.13 Slimey Inc. manufactures skin moisturizer. The graph of the cost function $C(x)$ is shown below. Cost is measured in dollars and x is the number of gallons of moisturizer.



(a) Sum What are the fixed costs for manufacturing the moisturizer?

(b) What is the slope of the graph of the cost function?

(c) What are the units for the marginal cost?
Circle one

i. dollars
ii. dollars per gallon
iii. gallons
iv. gallons per dollar

3.14 `[On Webwork]` The price-demand equation for avocados is

$$20p + x = 25,$$

where p is the price of an avocado and x is the weekly demand (in thousands) for avocados.

Write an expression for revenue as a function of the **price** of an avocado.

$R(p) =$

3.15 `[On Webwork]` The DingGnat Doorknob Company intends to sell a new line of square doorknobs. The price-demand function is $p(x) = 45.50 - .06x$. That is, $p(x)$ is the price at which x knobs that can be sold.

b. Sum How many knobs can be sold at a price of \$38.30?

a. Write an equation for the revenue function $R(x)$.

3.16 `[Practice only]` The DingGnat Doorknob Company intends to manufacture a new line of square doorknobs. The company spends \$2,250 dollars in fixed costs to set up the machines and an additional V dollars for each doorknob they make.

a. Write an equation for the cost function $C(x)$, where x is the number of knobs they make.

b. Sum If it costs \$4,850 to make 1000 knobs, what is the variable cost V?.

3.17 `[On Webwork]` The DingGnat Doorknob Company intends to manufacture a new line of square doorknobs. The company spends F dollars in fixed costs to set up the machines and an additional \$3.25 for each doorknob they make.

a. Write an equation for the cost function $C(x)$, where x is the number of knobs they make.

b. Sum If it costs \$4,400 to make 100 knobs, what are the fixed costs, F?

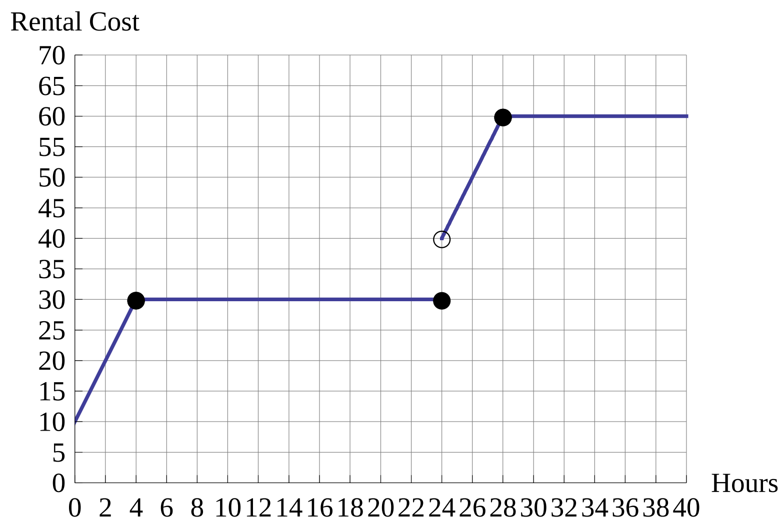
3.18 `[Done in 103L, variant on webwork.]` Sum The demand $D(p)$ for StarBoys Frapaccino is function of the price p of a serving. The price p is measured in dollars and the demand $D(p)$ is measured in thousands of servings per day. Translate the symbols $D(2.40) = 171$ into words.

3.19 `[Done in 103L, no webwork version]` Clippo Inc. manufactures and sells paper clips. The revenue $R(x)$ from the sales of paper clips is a function of how many x they sell. The number sold, x, is measured in thousands of papers clips. The revenue, $R(x)$, is measured in dollars.

a. Sum Write a summary in words for the statement $R(204) = 890$.

b. Clippo management wants to know how many paper clips they must sell to get \$14,000 in revenue. Translate this problem into symbolic form.

3.20 At Lake Landloch you can rent boats and the payment follows the graph below.



(a) What is the cost of renting a boat from 8:00am Monday morning to noon on the next day (Tuesday)?

Summary: It costs \$60.

(b) Fill in the blanks in the formula below for the cost $C(x)$ of renting a boat on the domains given below.

$$C(t) = \begin{cases} 10 + 5t, & \text{if } 0 \leq t \leq 4, \\ 30, & \text{if } 4 < t \leq \boxed{24}, \\ \boxed{\text{5t-80}}, & \text{if } \boxed{24} < t \leq 28 \\ 60, & \text{if } t \geq 28 \end{cases}$$

(c) What is the slope of the the graph $y = c(t)$ on the interval $[0, 4]$?

3.21 `[Done in 103L, no webwork version]` The Trussville Utiliities uses the rates shown in the table below to compute the monthly cost, $C(x)$, of natural gas for residential customers. Usage, x, is measure in cubic hundred feet (CCF) of natural gas.

Base charge	\$8.00
First 1000 CCF	\$0.04 per CCF
Over 1000CCF	\$0.07 per CCF

a. Sum Find the charge for using 250 CCF.

b. Sum Find an expression for the cost function $C(x)$ for usage under 1000 CCF.

c. Sum Find an expression for the cost function $C(x)$ for usage over 1000 CCF.

3.22 `[On Webwork]` The table below shows the electricity rates charged by Madison Utilities in the winter months. Usage, x, is measured in kilowatt hours (KWH) and the charge for using x KWHs is denoted by $C(x)$.

Base charge	\$8.50
First 700 KWH	\$0.06 per KWH
Over 700 KWH	\$0.17 per KWH

a. Sum Find the charge for using 900 KWH.

b. Sum Find an expression for the cost function $C(x)$ for usage under 700 KWH.

c. Sum Find an expression for the cost function $C(x)$ for usage over 700 KWH.

3.23 `[Done in 103L, no webwork version]` The cost to produce x memory chips is

$$C(x) = 800 + 5.25x,$$

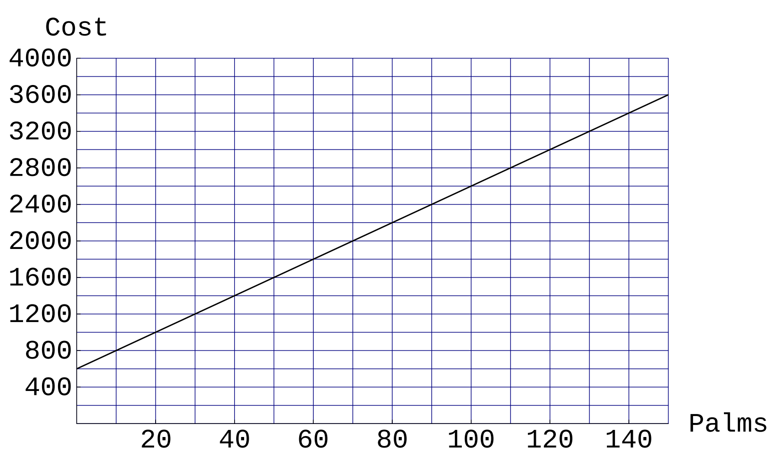
where cost is measured in dollars.

(a) Sum What is the cost to produce 100 memory chips?

(b) Sum What is the average cost (per chip) to produce 100 memory chips?

(c) Sum Write a formula for the average cost (per chip) $\overline{C}(x)$ to produce x memory chips.

3.24 `[On Webwork]` A nursery grows palm trees from seeds. After a seed has grown for two years, the palm tree is ready to sell. The graph of the cost function is shown below. Cost is given in dollars.



(a) Sum Estimate the cost to grow 100 palms from the graph.

(b) Sum Estimate the average cost (per palm) to grow 100 palms.

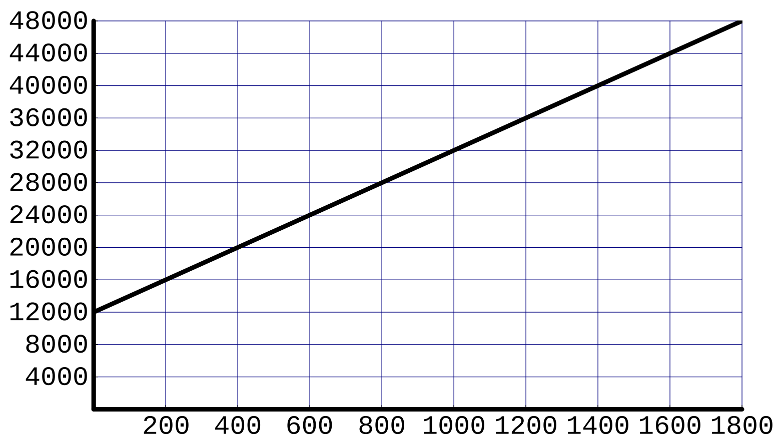
3.25 `[Practice only]` Bigelow Security Inc. is considering producing and selling a new kind of car alarm. The research department estimates that the fixed costs to retool and manufacture the car alarms will be $12,000 and the variable costs will be $20 per alarm.

(a) Write an algebraic expression for the total cost to produce x alarms:

Complete Solution:
The fixed costs are 12000 and the variable costs are 20 per unit. So

$$C(x) = 12000 + 20x.$$

(b) Draw an accurate graph of the cost function.



(c) The price demand function of the car alarms is

$$p = 340 - (0.50)x.$$

Price is given in dollars, and x is the demand at price p.
Write an algebraic expression for the revenue function, $R(x)$.
Complete Solution:
$R(x) = xp = x(340 - 0.50x)$.

3.26 `[On Webwork]` Sally sells cookies in front of her house. To make 12 cookies, it costs her \$3.00. To make 24 cookies, it costs \$5.00.

(a) Draw a set of axes plot and label the two points on the graph below, label the axes, and draw a line showing cost as a function of the number of cookies made.

(b) Calculate the slope of the line.

(c) Write an equation for the cost function in slope-intercept form:

$$C(x) =$$

$C(x)$ denotes the cost to make x cookies.

(d) Sum Using either your equation or your graph, find Sally's fixed cost.

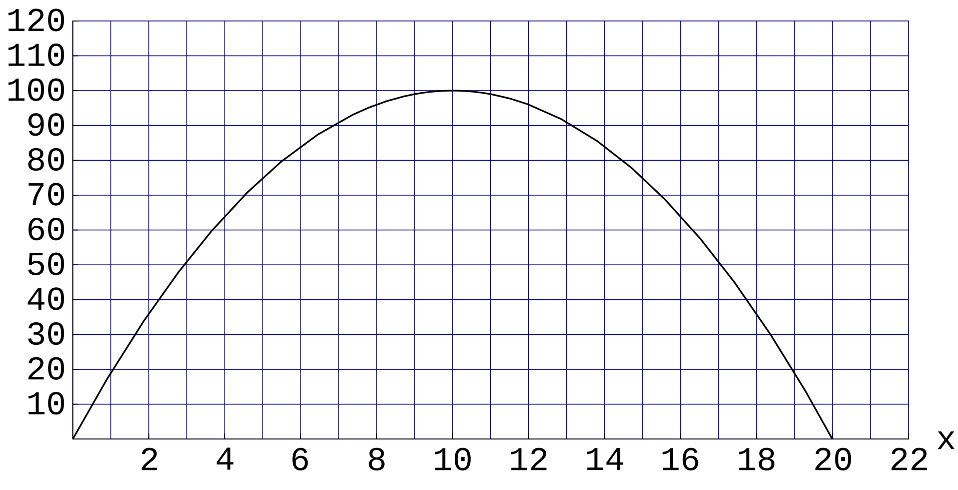
(e) Sum What is the variable cost of making one cookie?

4 Section 2.2: Elementary functions: Graphs and Transformations

4.1 `[Done in 103L, no webwork version]` A company manufactures and sells x wigits per week. The weekly price-demand and cost functions are:

$$\begin{aligned} p(x) &= 20 - x \\ C(x) &= 25 + 5x. \end{aligned}$$

The revenue function $R(x)$ is graphed below:



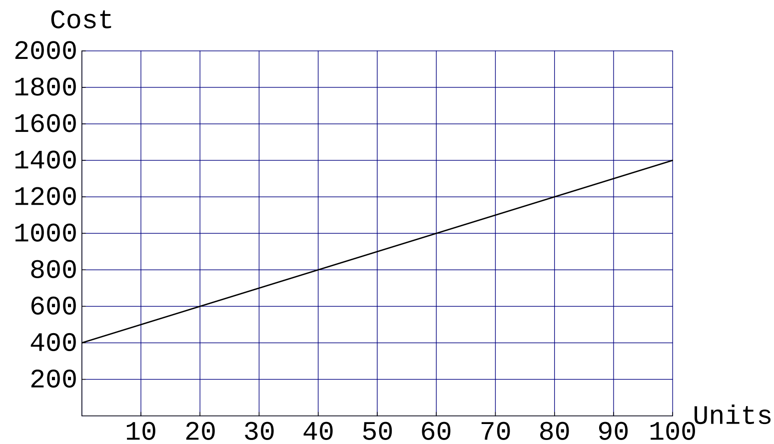
(a) Write an expression for the revenue function, $R(x)$.

(b) Graph the cost function, $C(x)$, on the graph above.

(c) Mark the bread-even points on the graph.

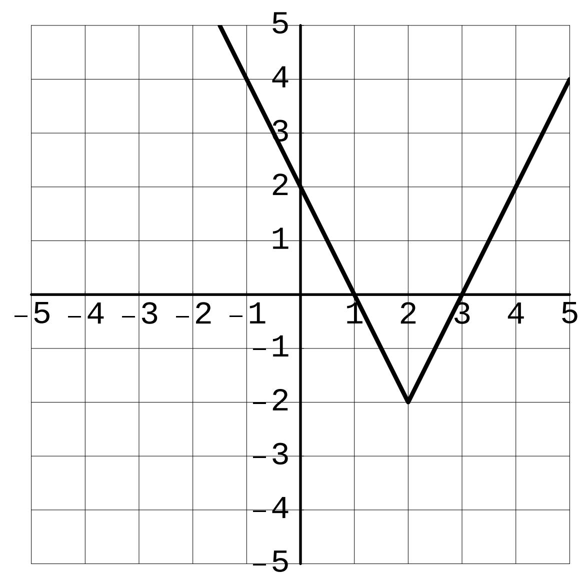
(d) Shade the region where the company makes a profit.

4.2 `[Done in 103L, variant on webwork.]` Sum The graph of a cost function is show below. The cost $C(x)$ to produce x units has two parts: the fixed cost F, and the variable (per unit) cost V. Determine the values of F and V from the graph. Explain how you found F and V from the graph.



4.3 `[Done in 103L, variant on webwork.]` The graph of an absolute-value function $f(x)$ is shown below.

Find the y-intercept	
Find the x-intercepts	
Find $f(2)$	



4.4 `[Done in 103L, variant on webwork.]` Relative to the graph of

$$y = 3|x| + 1,$$

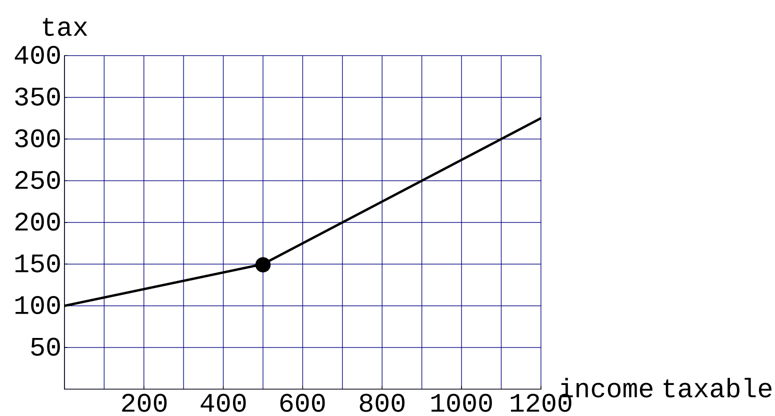
the graphs of the following equations have been changed in what way?

Answer ↓

Answer	
	1. $y = 3|x - 5| + 1$
	2. $y = (3/5)|x| + (1/5)$
	3. $y = 3|x| + 6$

A	shifted 5 units right
B	shifted 5 units left
C	stretched vertically by a factor of 5
D	shrinked vertically be a factor of 1/5
E	shifted 5 units up
F	shifted 5 units down

4.5 `[Done in 103L, variant on webwork.]` The following graph shows the amount of tax $T(x)$ (in dollars) for a taxable income of x (in dollars).



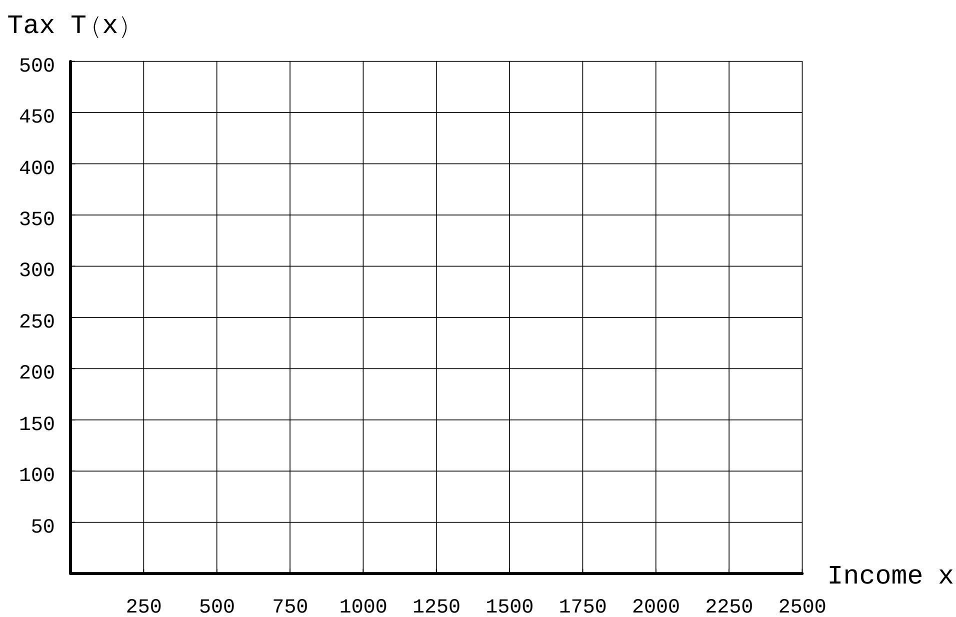
Sum What is the tax rate for incomes over \$500? Give your answer as a percentage and explain how you calculated it.

4.6 `[Done in 103L, no webwork version]` There is an income tax on the planet Bozone. Both annual income, x, and income tax, $T(x)$, are measured in the local currency, the Bozat ($\mathfrak{B}$). The Bozonian tax table is shown below.

Between	But Not Over	Base Tax	Rate	Of the Amount Over
𝔅0	𝔅1,000	0	10%	𝔅0
𝔅1,000	𝔅2,500	𝔅100	20%	𝔅1,000

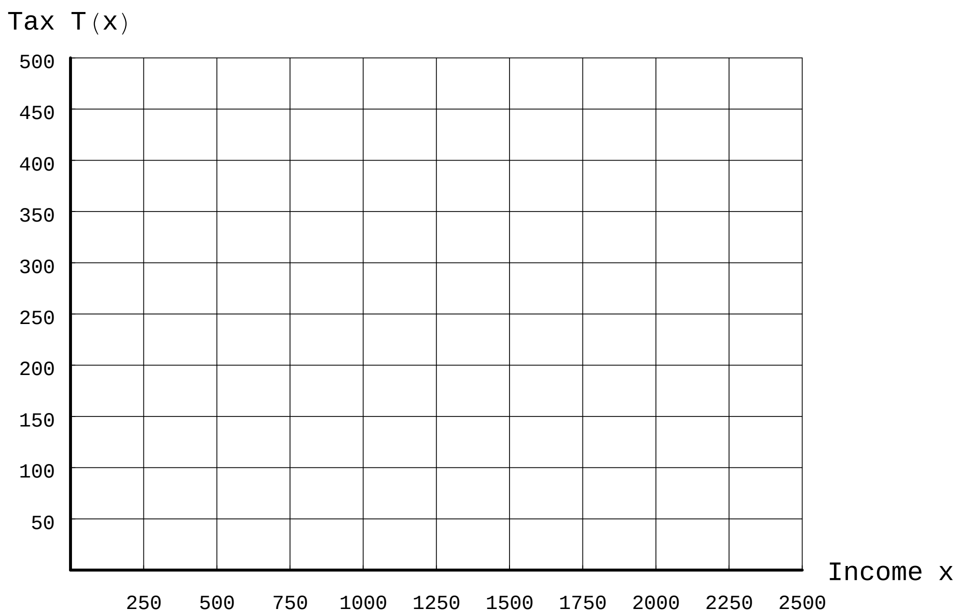
(a) The equation for the income tax on income between 𝔅1,000 and 𝔅2,500 is of the form $T(x) = mx + b$. Find the values of m and b.
$m =$ ______, $b =$ ______

(b) Draw an accurate graph of the tax function $T(x)$.



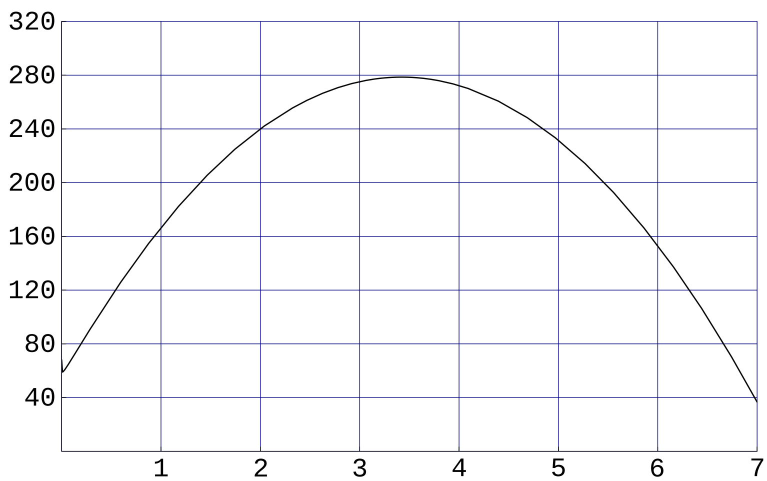
4.7 `[Done in 103L, variant on webwork.]` (will need modification try doing it as in part (a) above for both income brackets) There is an income tax on the planet Bozone. Both annual income, x, and income tax, $T(x)$, are measured in the local currency, the Bozat (𝔅). If the annual income $x < 1500$, then the income tax is 10% of the income: $T(x) = .10x$. If annual income $x \geq 1500$, then the income tax is 20% of the income: $T(x) = .20x$.

(a) Draw an accurate graph of the tax function $T(x)$.



(b) Sum Is the income tax function, $T(x)$, continuous? Explain.

4.8 `[On Webwork]` Sum The graph below shows the amount of $A(t)$ electricity used in the city of Roseville as a function of the time of day t. The unit of measurement for electricity is megawatts and the time t is the number of hours past noon. How much electricity is being used at 2:00pm?



5 Section 2.3: Quadratic Functions, General Polynomials, Rational

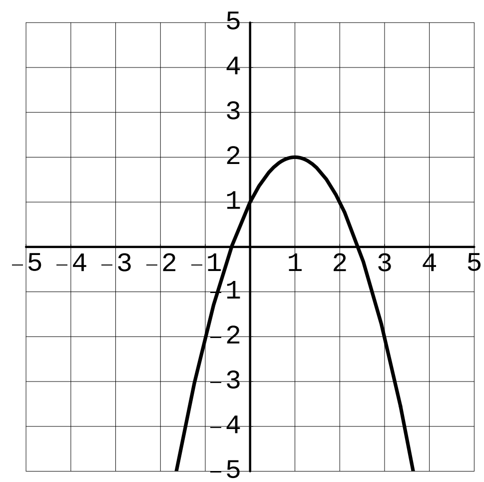
5.1 `[On Webwork]` Let $f(x)$ be the quadratic function:

$$f(x) = 3x^2 + 6x - 1.$$

	Answer
Write $f(x)$ in the vertex-form $f(x) = a(x-h)^2 + k$.	
What are the coordinates of the vertex of the parabola?	
Find the y-intercept	

5.2 `[Done in 103L, no webwork version]` The graph of a quadratic function $f(x)$ is shown below.

Find the vertex of the parabola:	
Find $f(2)$	
Write an equation for $f(x)$	
$f(x) =$	



5.3 `[Done in 103L, variant on webwork.]` Let $f(x)$ be the quadratic function:

$$f(x) = 2x^2 - 8x + 2.$$

(a) By completing the square, write $f(x)$ in the vertex-form.

(b) What is the vertex of the parabola?

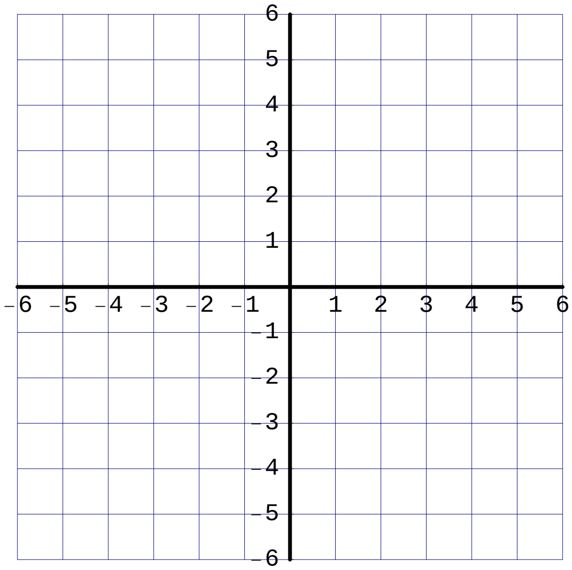
(c) What is the maximum or minimum value of the function?

(d) What is the range of the function?

(e) What is the y-intercept?

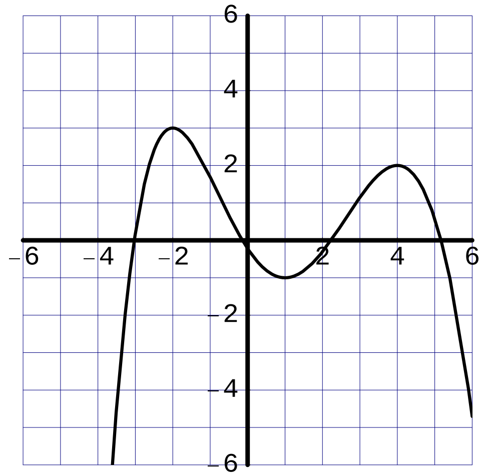
(f) Does the parabola have one, two, or no x-intercepts?

5.4 `[On Webwork]` Draw an accurate graph of the function $f(x) = \dfrac{4x}{2x+1}$. Your graph should clearly show the asymptotes, the point $(-1, f(-1))$, and the y-intercept.



5.5 `[Done in 103L, variant on webwork.]` Find the coordinates of the turning points in the graph below. Identify each turning point as either a local maximum or a local minimum.

Turning point coordinates	Local max or min?



5.6 `[Done in 103L, no webwork version]` Consider the polynomial function

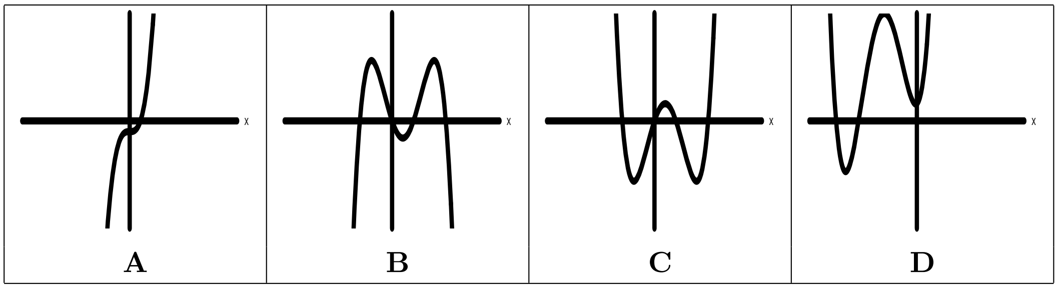
$$f(x) = 2x^4 - 3x^2 + 4x - 7.$$

(a) What is the degree of this polynomial?

(b) What is the maximum number of times this polynomial can intersect the x-axis?

(c) What is the maximum number of turning points this polynomial can have?

5.7 `[On Webwork]` Let $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ be a polynomial function of degree 4, where a is positive and the function has four x-intercepts. Which one of the graphs below could be the graph of $y = f(x)$? Why?



5.8 `[Done in 103L, variant on webwork.]` Relative to the graph of

$$y = \frac{1}{x+1},$$

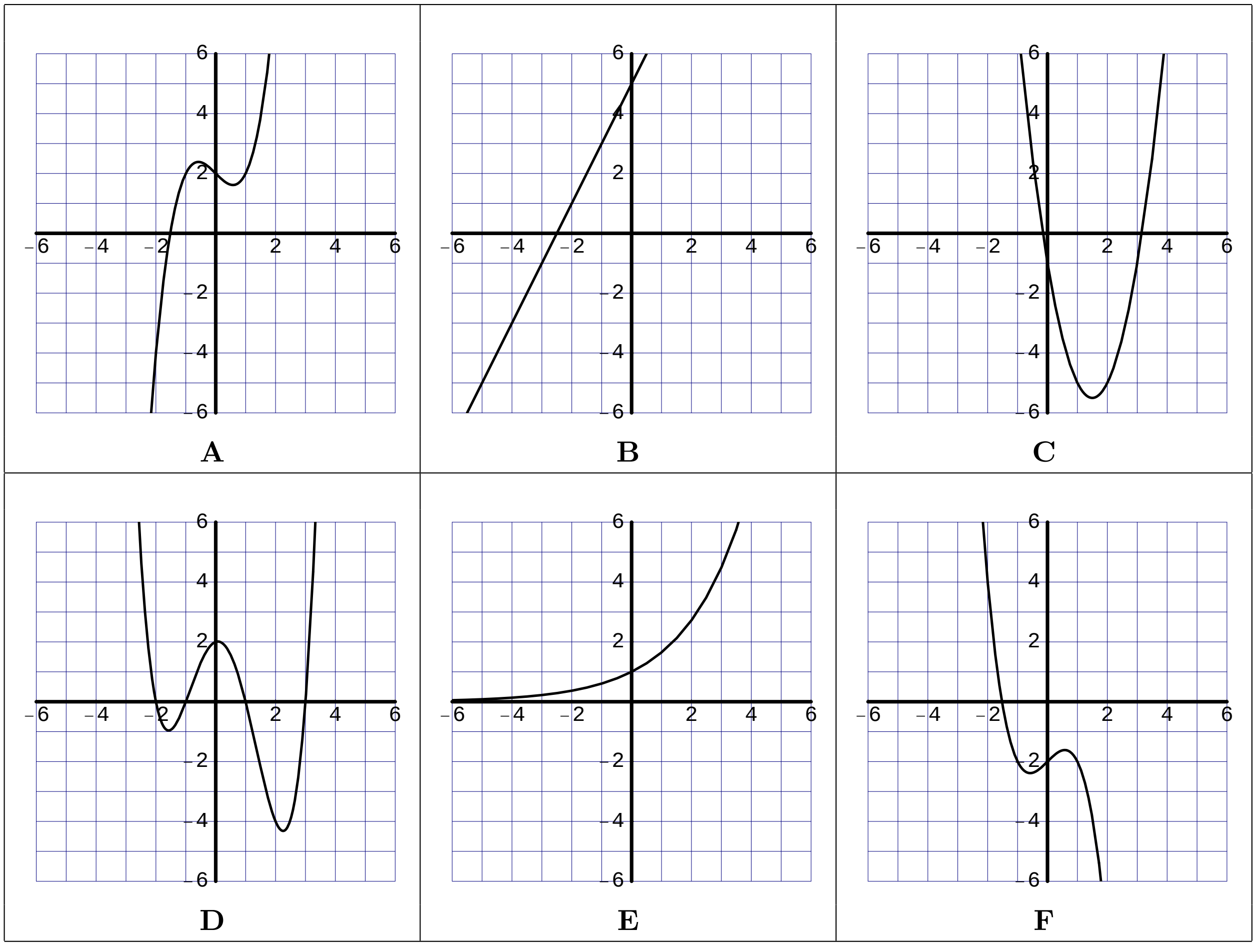
the graphs of the following equations have been changed in what way?

Answer
↓

Answer	
	1. $y = \dfrac{5}{x+1}$
	2. $y = \dfrac{1}{(x+5)+1}$
	3. $y = \dfrac{1}{x+1} - 5$

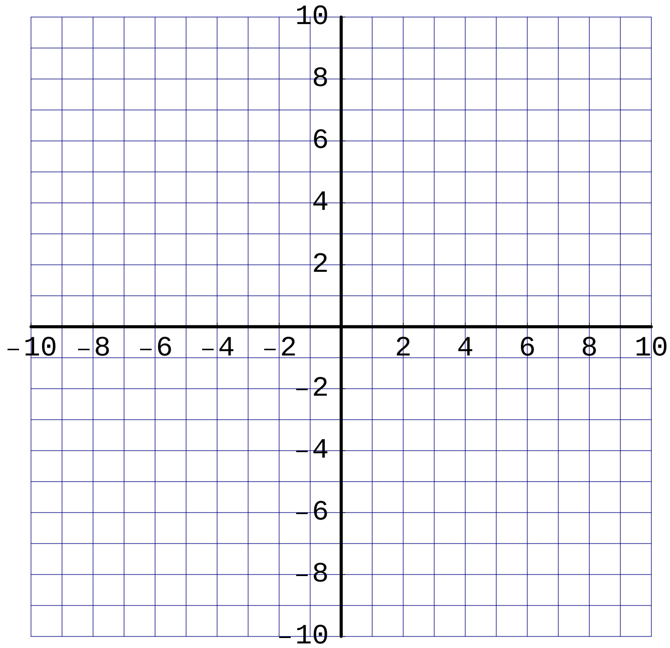
A	shifted 5 units right
B	shifted 5 units left
C	stretched vertically by a factor of 5
D	shrinked vertically be a factor of 1/5
E	shifted 5 units up
F	shifted 5 units down

5.9 `[Done in 103L, variant on webwork.]` Match the graph with function:



Graph	Function
	$f(x) = 2x^2 - 6x - 1$
	$f(x) = -x^3 + x - 2$
	$f(x) = (1/3)(x-1)(x+2)(x-3)(x+1)$
	$f(x) = x^3 - x + 2$

5.10 `[Practice only]` Make an **accurate** graph of the function $f(x) = 2(x-1)^2 - 4$. Mark the y-intercept, the vertex, and the points $(-1, f(-1)), (3, f(3))$ with dots on the graph.



5.11 `[On Webwork]` Let $f(x)$ be the quadratic function:

$$f(x) = -x^2 + 6x + 1.$$

	Answer
Write $f(x)$ in the vertex-form $f(x) = a(x-h)^2 + k$.	
What are the coordinates of the vertex of the parabola?	
Does the parabola open up or down?	

5.12 `[Practice only]` Make an **accurate** graph of the function $f(x) = -(x-2)^2 + 5$. Mark the y-intercept, the vertex, and the points $(1, f(1)), (5, f(5))$ with dots on the graph.

6 Exponential functions

6.1 see webwork

7 Logarithmic functions

7.1 see webwork

8 Simple and Compound Interest

8.1 `[Done in 103L, variant on webwork.]` An investment account earns 10% compounded quarterly. An initial investment of \$7,000 (present value) grows to \$14,000 (future value) in t years. Then $t = \dfrac{\log a}{b \log c}$. Find a, b, c.

$a =$ ____________, $b =$ ____________, $c =$ ____________

8.2 `[Done in 103L, variant on webwork.]` \$5,000 is invested in an account that pays 6% compounded quarterly. The amount in the account after 20 years is $P(1+a)^b$. Find the values of P, a, b:

$P =$ ____________, $a =$ ____________, $b =$ ____________

8.3 (10 pts) In the problems below please simplify your answers as far as possible without a calculator. You may leave your answers in terms of exponentials and logarithmic expressions. Suppose we deposit \$20,000 into an investment account.

(a) How much will our account have after 12 years if it is invested at an annual interest rate of 3% compounded every four months?

What are the correct units in your answer:

(b) How long will it take for the investment account to grow to \$100,000 if annual interest is 3% and it is compounded continuously?

What are the correct units in your answer:

8.4 `[Done in 103L, variant on webwork.]` Suppose we invest \$300.

(a) [Sum] What amount will our account have after 7 years if it earns an annual rate of 3% compounded daily?

(b) [Sum] How long will it take for our account to grow to \$1000 if it is invested at an annual rate of 3% compounded continuously?

8.5 `[Done in 103L, variant on webwork.]` Suppose we deposit \$7000 into an investment account. Simplify you answers as far as possible without a calculator. You may leave your answers in terms of exponentials and logarithmic expressions.

(a) What amount will our account have after 15 years if it is invested at an annual rate of 5% compounded quarterly.
Complete Solution:
We use the formula for future value:

$$A = P(1+i)^n,$$

where $P = 7000, i = .05/4, n = 15 \times 4$.

$$A = 7000(1.0125)^{60}.$$

Summary: The account will have $7000(1.0125)^{60}$ dollars after 15 years.

(b) What annual rate of interest is needed in order for the investment account to grow from \$7000 to \$14,000 in 10 years if interest is compounded continuously?

Complete Solution:
We use the formula for continuous compounding:

$$A = Pe^{rt},$$

where $P = 7000, A = 14000, t = 10$ and r is unknown. Thus we must solve

$$14000 = 7000e^{10r}$$

for r:

$$\begin{aligned} 14000 &= 7000e^{10r} \\ e^{10r} &= 2 \\ 10r &= \ln 2 \\ r &= \frac{\ln 2}{10}. \end{aligned}$$

Summary: For an investment of \$7000 to grow to \$14000 in 10 years with continuous compounding, the rate must be $r = (\ln 2)/10$.

8.6 `[On Webwork]` Sum We deposit \$2,000 into an account earning 6% interest compounded semiannually. How many years will it take for the account grows to \$5,000?

9 Section 10.1: Limits

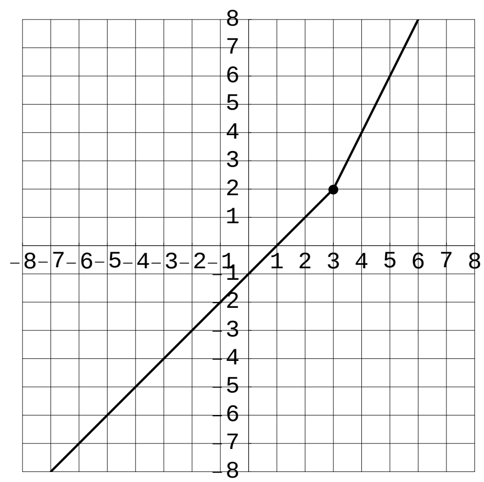
9.1 see webwork

10 Section 10.2: Continuity

10.1 `[Done in 103L, no webwork version]` Consider the following function.

$$f(x) = \begin{cases} x - 1, & \text{if } x \leq 3 \\ 2x - 4, & \text{if } x > 3 \end{cases}$$

(a) Sketch a graph of $y = f(x)$.
Complete Solution:



(b) Where is this function continuous? Explain why using limits.
Complete Solution:
The function is continuous at every value of x. The only possible exception is at $x = 3$ and at that point,

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} x - 1 = 2,$$

and

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} 2x - 4 = 2.$$

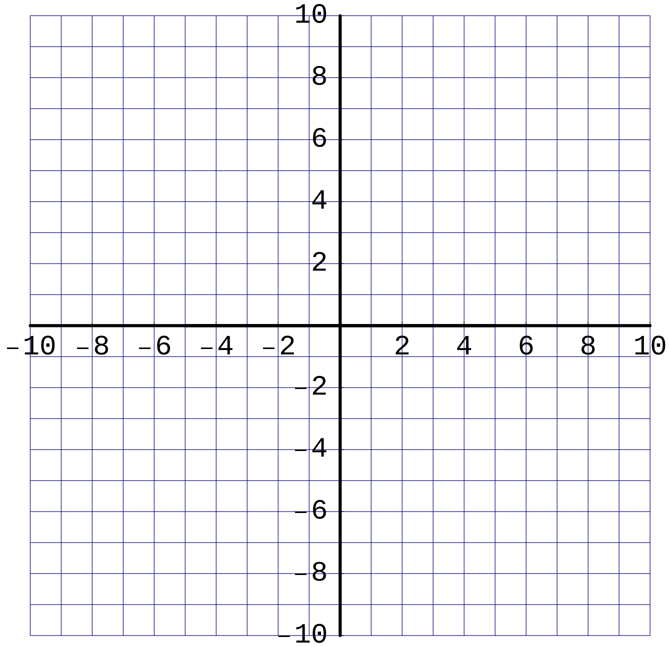
So $\lim_{x \to 3} f(x)$ exists and equals the value of the function $f(3) = 2$.

(c) Where is this function differentiable? (No explanation necessary)
Complete Solution:
The function is differentiable at every value of x except at $x = 3$. The function is not differentiable at $x = 3$.

10.2 `[Done in 103L, variant on webwork.]` Let $f(x)$ be the following piecewise defined function:

$$f(x) = \begin{cases} 2x + 3, & \text{if } x \leq 0 \\ 3, & \text{if } 0 < x < 4 \\ -x + 9, & \text{if } x \geq 4 \end{cases}$$

(a) Graph the function $y = f(x)$.



(b) Find $\lim_{x\to 0^+} f(x)$.

(c) Find $\lim_{x\to 4^+} f(x)$.

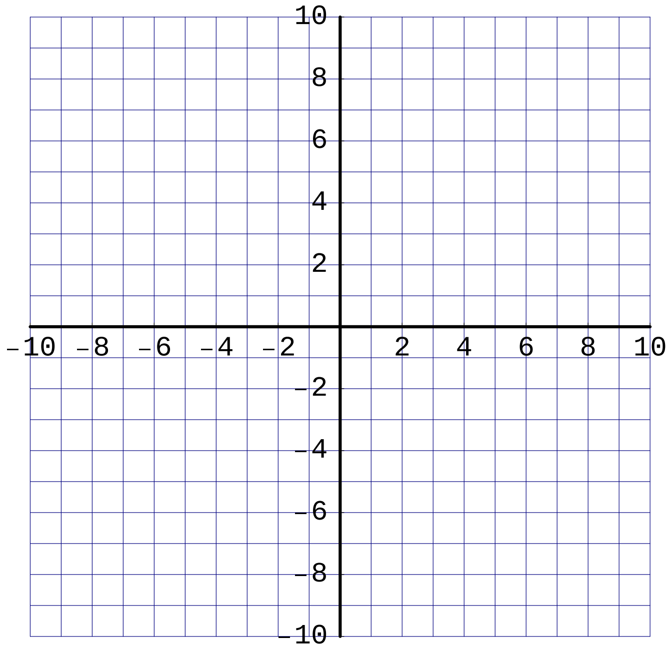
(d) Where is this function continuous?

(e) Where is this function differentiable?

10.3 `[Done in 103L, variant on webwork.]` Let $f(x)$ be the following piecewise defined function:

$$f(x) = \begin{cases} 2, & \text{if } x \le 0 \\ 3x - 2, & \text{if } 0 < x < 3 \\ x - 4, & \text{if } x \ge 3 \end{cases}$$

(a) Graph the function $y = f(x)$.



(b) Find $\lim_{x\to 0^+} f(x)$.

(c) Find $\lim_{x\to 4^+} f(x)$.

(d) Where is this function continuous?

(e) Where is this function differentiable?

11 Section 10.3: Infinite limits and limis to infinity

11.1 see webwork

12 Section 10.4: The derivative

12.1 `[Done in 103L, no webwork version]` Consider the revenue function $R(x) = 250x - x^2$ for producing x widgets.

(a) [Sum] Find the change in revenue when production changes from $x = 10$ to $x = 20$.

(b) [Sum] Find the average rate of change of revenue for this change in productions levels.

(c) [Sum] Use this to estimate the revenue at a production of $x = 21$.

12.2 `[On Webwork]` The profit (in dollars) from the sale of x palm trees is given by

$$P(x) = 20x - 0.01x^2 - 100.$$

(a) Sum Find the average change in profit if sales changes from 10 trees to 15 trees.

(b) Sum Find the profit and the instantaneous rate of change of profit at a sales level of 10 trees.

12.3 `[On Webwork]` Use the definition of the derivative to find the derivative of $f(x) = 2x^2 - 3$. Here are some steps.

(a) Find
$$f(x+h)$$

(b) Find
$$\frac{f(x+h) - f(x)}{h}$$

(c) Find
$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

12.4 `[Done in 103L, no webwork version]` The revenue from the sale of x cellphone towers is given by
$$R(x) = 1000x - 10x^2.$$
The derivative of the revenue function is given by $R'(x) = 1000 - 20x$.

(a) Sum What is the change in revenue if production is changed from $x = 5$ to $x = 6$ cellphone towers?

(b) What is the (instantaneous) rate of change in revenue at $x = 5$?

12.5 `[On Webwork]` The revenue from the sale of x high end cameras is given by
$$R(x) = 1000x - 5x^2.$$

(a) Sum What is the change in revenue if production is changed from $x = 10$ to $x = 11$ cellphone towers?

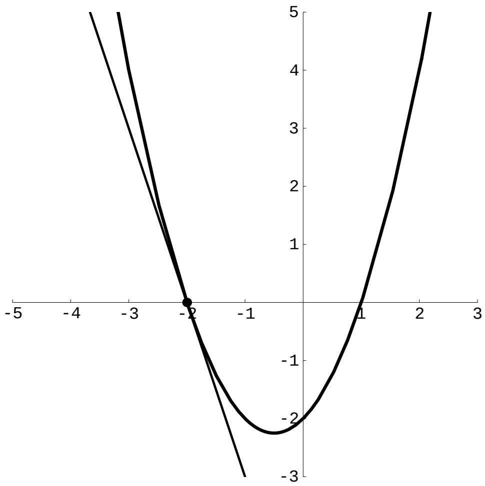
(b) What is the (instantaneous) rate of change in revenue at $x = 10$?

13 Section 10.5: Basic Derivatives

13.1 `[On Webwork]` Let $f(x) = -2x^2 + x + 1$. Find the equation of the line tangent to the graph of $y = f(x)$ at the point $(-3, f(-3))$.

13.2 `[Done in 103L, no webwork version]` Let $f(x) = 2x^2 - 3x + 4$. Find the equation of the line tangent to the graph of $y = f(x)$ at the point $(0, f(0))$.

13.3 `[Done in 103L, variant on webwork.]` Let $f(x) = x^2 + x - 2$. Find the equation of the line tangent to the graph of $f(x)$ at the point $(-2, f(-2))$ shown below.

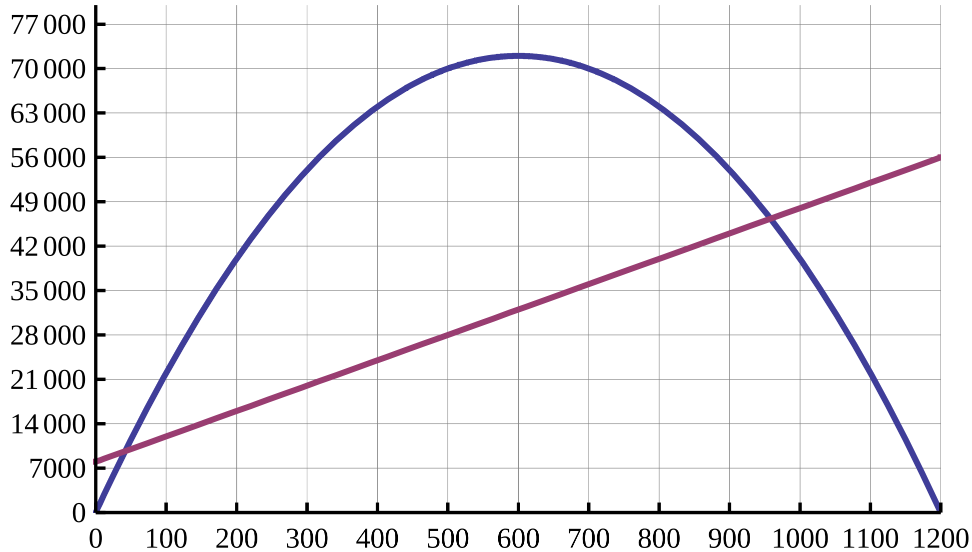


13.4 `[On Webwork]` Let $f(x) = -x^2 - 4x$. Find the equation of the line tangent to the graph of $f(x)$ at the point shown below.

14 Section 10.7: Marginal Analysis

14.1 at least one like old webwork

14.2 `[On Webwork]` The graphs of the revenue and cost functions for the production and sale of x units are shown below. The cost function is the straight line and the revenue function is the curve.



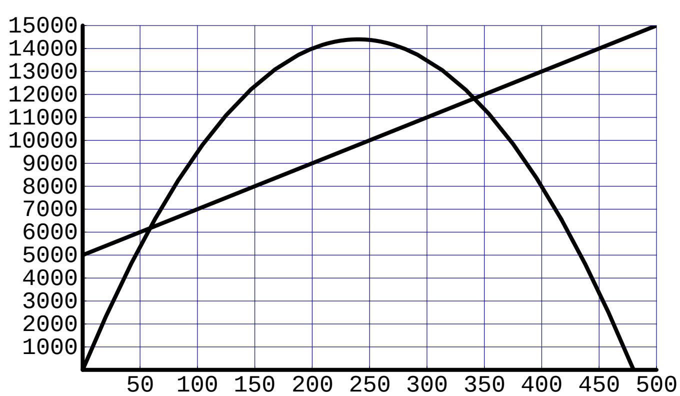
(a) Use the graph to estimate the production level x that maximizes the profit.

(b) Mark the points $(x, C(x))$ and $(x, R(x))$ on the graphs of the cost and revenue functions corresponding to the value of x that maximizes profit.

(c) What is the maximum profit?

(d) If the fixed costs increase by ten dollars, should the production level be raised, lowered, or remain the same to maximize profit? Explain in terms of the graph.

14.3 `[On Webwork]` The graphs of the revenue and cost functions for the production and sale of x units are shown below. The cost function is the straight line and the revenue function is the curve.

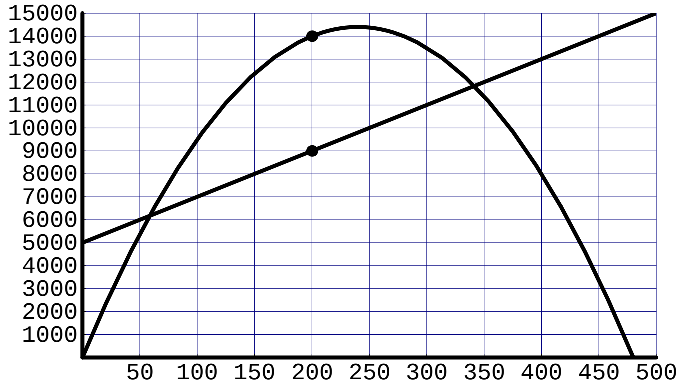


(a) Use the graph to estimate the production level x that maximizes the profit.

(b) Mark the points $(x, C(x))$ and $(x, R(x))$ on the graphs of the cost and revenue functions corresponding to the value of x that maximizes profit.

Complete Solution:

The maximum profit occurs at the production level x where marginal revenue equals marginal cost. That is, where the slopes of the revenue and cost curves are equal. The slopes are equal at $x = 200$. The points $(200, C(200)) = (200, 9000)$ and $(200, R(200)) = (200, 14000)$ are marked below.



Summary: Profit is maximized when the production level is 200 units.

(c) What is the maximum profit?

Complete Solution:

Profit is the difference between revenue and cost. $R(200) - C(200) = 14000 - 9000 = 5000$.

Summary: The maximum profit is \$5000.

(d) If the cost per unit decreases, should the production level be raised or lowered to maximize profit? Explain in terms of the graph.

Summary: If the cost per unit decreases, then the slope of the cost curve decreases. The slope of the revenue curve decreases as production level increases. So production level must be increased (raised) to maximize profit.

14.4 `[On Webwork]` Sequoia Publishing Company plans to publish a vegetarian cookbook. The cost (in dollars) to produce x books is

$$C(x) = 2600 + 10x.$$

The price-demand equation is

$$p = 80 - (0.14)x$$

and the revenue function is

$$R(x) = 80x - (0.14)x^2.$$

(a) Compute the marginal cost and marginal revenue functions:

Complete Solution:

Marginal cost: $C'(x) = 10$

Marginal revenue: $R'(x) = 80 - 0.28x$

(b) Use the marginal revenue function to approximate the revenue for selling the 201st book. The revenue for selling the 201st book is approximately $R'(200)$:

$$R'(200) = 80 - 0.28(200) = 24.00.$$

Summary: The revenue for selling the 201st book is approximately \$24.00.

14.5 [On Webwork] A company manufactures and sells x wigits per week. The weekly price-demand function is:

$$p(x) = 20 - x.$$

(a) Find the marginal revenue function.

(b) Sum Use marginal revenue to estimate the additional revenue earned by producing 6 wigits instead of 5 wigits.

14.6 [On Webwork] The price-demand equation for the sale of Atomic TV sets is

$$p + 0.8x = 500.$$

The price p is in dollars, and x is the demand for Atomic TVs at a price of p dollars.

(a) Find the revenue function $R(x)$ as a function of the demand, x.

(b) Sum Find the marginal revenue at $x = 100$ and write a sentence explaining what this means in terms of TV sales.

14.7 [On Webwork] The profit function from manufacturing and selling x BabCo Lounge Chairs is

$$P(x) = 40x - 140 - (0.1)x^2.$$

(a) Sum Find the exact additional profit for manufacturing and selling 11 chairs instead of 10 chairs.

(b) Find the marginal profit at $x = 10$.

14.8 [On Webwork] Acme Office Supplies manufactures file cabinets. The cost (in dollars) of producing x file cabinets is given by

$$C(x) = 1020 + 50x - x^2.$$

(a) Sum Find the exact additional cost of producing 8 file cabinets instead of 7.

(b) Find the marginal cost function.

(c) Sum Use the marginal cost function to approximate the additional cost of producing the 8 file cabinets instead of 7.

14.9 `[Practice only]` Sum The price-demand function for the sale of yo-yos is

$$p = 5.50 - .01x,$$

where p is the price of a yo-yo in dollars, and x is the demand for yo-yos at a price of p dollars. A simple calculation shows that $R'(290) = -.30$. Write a sentence explaining what this means in terms of the yo-yo problem. Be sure to use the correct units for $R'(290)$.

15 Section 11.3-4: Product, quotient, generalized power rules

15.1 `[On Webwork]` Let $f(x) = (-x^2 + x + 1)^4$.

(a) Find the derivative $f'(x)$.

(b) Find $f'(1)$.

15.2 Find the derivatives of the following functions and simplify.

(a) $s(x) = (3x - 5x^3)^{1/2} + 100$

(b) $r(x) = \dfrac{5x - 6}{3x + 4}$

Answer only:
$s'(x) = \frac{1}{2}(3x - 5x^3)^{-1/2}(3 - 15x^2)$ and $r'(x) = \frac{38}{(3x+4)^2}$

15.3 `[Done in 103L, variant on webwork.]` Let $f(x) = -x^2 + 12x + 1$.

(a) Find the derivative $f'(x)$.

(b) Find $f'(-1)$.

15.4 Bling & Co. market faux-diamond studded coffee cups to the local market in Pasadena. Suppose that the number of coffee cups that people are willing to buy per week at a price of p dollars per cup is given by the equation

$$f(p) = \frac{p + 1}{p^2 + 2p + 2}$$

(a) Find $f'(p)$. Simplify your answer.
Complete Solution:

$$\begin{aligned} f'(p) &= \frac{(1)(p^2+2p+2)-(2p+2)(p+1)}{(p^2+2p+2)^2} \\ &= \frac{p^2+2p+2-(2p^2+4p+2)}{(p^2+2p+2)^2} \\ &= \frac{-p^2-2p}{(p^2+2p+2)^2} \end{aligned}$$

(b) Is weekly demand increasing, decreasing or neither at a price of $1 per coffee cup? Why?

$$f'(1) = \frac{-(1)^2-2(1)}{((1)^2+2(1)+2)^2}$$

which is clearly negative.

> **Summary:** Since the derivative is negative at a price of $1 per cup we see that demand is decreasing.

15.5 `[Done in 103L, no webwork version]` Find the derivatives of the following functions and simplify.

(a) $f(x) = -(x-2)^2+3$

(b) $s(x) = 3x^2+5x+100$

(c) $r(x) = \dfrac{3x-2}{(2x+5)^2}$

Complete Solution:

$$\begin{aligned} f'(x) &= -2(x-2) \\ s'(x) &= 6x+5 \\ r'(x) &= \frac{3(2x+5)^2-2(2x-5)^1(2)(3x-2)}{(2x-5)^4} \\ &= \frac{3(2x+5)-4(3x-2)}{(2x-5)^3} \\ &= \frac{6x+15-12x+8}{(2x-5)^3} \\ &= \frac{-6x+23}{(2x-5)^3}. \end{aligned}$$

15.6 `[On Webwork]` Find the derivative of the function

$$f(x) = (x^2 + 3x + 1)(14 - 3x^2).$$

15.7 `[On Webwork]` Let $f(x) = (x^2 - x + 1)^3$.

a. Find the derivative $f'(x)$.

b. Find $f'(1)$.

15.8 `[On Webwork]` Find the derivative of the function

$$f(x) = (x^3 + 4x + 1)(150 - 3x).$$

15.9 `[On Webwork]` Find the derivative of the function

$$f(x) = \sqrt{5x + 3}.$$

15.10 `[On Webwork]` Find the derivative of the function

$$f(x) = \sqrt{x} - \frac{1}{x^3}.$$

15.11 `[On Webwork]` Find the derivative of the function

$$f(x) = \frac{2x - 1}{3x + 5}.$$

15.12 `[Done in 103L, no webwork version]` If $R(x) = xp(x)$ what is $R'(x)$ in terms of $p(x)$ and $p'(x)$? If $P(x) = R(x) - C(x)$, express $P'(x)$ in terms of $R'(x)$, and $C'(x)$

16 Section 11.7: Elasticity

16.1 `[On Webwork]` A company manufactures and sells x clocks per week with weekly demand function: $f(p) = 20 - 2p$ where p is the price per clock.

(a) Compute the elasticity of demand function for this demand function.
Complete Solution:

$$E(p) = \frac{-pf'(p)}{f(p)} = \frac{2p}{20-2p}.$$

(b) At $p = \$8$: a price increase of 10% will create a demand decrease of what percent?
Complete Solution:
Elasticity at $p = 8$ is $E(8) = 4$. Thus the relative rate of decrease in demand is approximately 4 times the relative rate of increase in price.

Summary: Demand is will decrease 40%.

16.2 `[On Webwork]` The demand equation p is given by

$$x + p = 4800.$$

(a) Write demand as a function of price.

(b) Find the elasticity of demand at a price of \$800?

(c) Sum If the price increases 10% from a price of \$800, what is the approximate (percentage) change in demand? State whether demand will increase or decrease.

16.3 `[On Webwork]` The demand function at a price p is given by

$$f(p) = 3000 - 2p.$$

(a) Find the elasticity of demand.

(b) Sum Is the elasticity of demand at a price of 600 elastic, inelastic, or unitary? Explain.

16.4 `[Done in 103L, no webwork version]` The demand function at a price p is given by

$$f(p) = 4000 - 2p.$$

(a) Find the elasticity of demand.

(b) Sum At what price is elasticity of demand unitary?

17 Section 12.5: Absolute Max/Min

17.1 `[On Webwork]` Sum Let $f(x) = x^3 - 27x$. If they exist, find the absolute maximum and the absolute minimum of $f(x)$ on the intervals below. Give explanations and if an absolute max or min does not exist, then say why.

(a) $(-\infty, \infty)$

(b) $[-3, \infty)$

(c) $[-3, 0]$

(d) $[0, 10]$

18 Section 12.6: Optimization

18.1 `[On Webwork]` AmeriCam manufactures and sells motion picture cameras. The price demand equation is

$$p = 2800 - 25x,$$

where p is the price (in dollars) at which x cameras can be sold.

(a) What is the *demand* if the price is \$1000 ?
Complete Solution:
If $p = 1000$, then x must satisfy the equation

$$1000 = 2800 - 25x.$$

Thus

$$x = \frac{2800 - 1000}{25} = 72.$$

Summary: At a price of \$1000 each, the demand is 72 cameras.

(b) `[On Webwork]` The cost to produce x cameras is given by

$$C(x) = 14500 + 800x.$$

and the revenue function is

$$R(x) = x(2800 - 25x).$$

How many cameras should be manufactured and sold to maximize profit?

Complete Solution:
Method 1: Profit is revenue minus cost so

$$P(x) = 2800x - 25x^2 - 14500 + 800x = -25x^2 + 2000x - 14500.$$

The graph of $y = P(x)$ is is a downward facing parabola as the coefficient of the x^2 term is negative. Thus there is a max at the vertex. To find the vertex set $P'(x) = 0$ and solve for x. Well

$$P'(x) = -50x + 2000$$

so,

$$\begin{aligned} P'(x) &= 0 \\ -50x + 2000 &= 0 \\ -50x &= -2000 \\ x &= 40. \end{aligned}$$

Summary: To maximize profit, AmeriCam should manufacture and sell 40 cameras.

OR Method 2: Profit is maximized at the production level, x, where marginal revenue equals marginal cost.

$$\begin{aligned} C'(x) &= 800 \\ R'(x) &= 2800 - 50x. \end{aligned}$$

So we must solve $C'(x) = R'(x)$ for x:

$$\begin{aligned} C'(x) &= R'(x) \\ 800 &= 2800 - 50x \\ 50x &= 2000 \\ x &= 40. \end{aligned}$$

Summary: To maximize profit, AmeriCam should manufacture and sell 40 cameras.

(c) What price should AmeriCam charge for each camera to maximize profit?

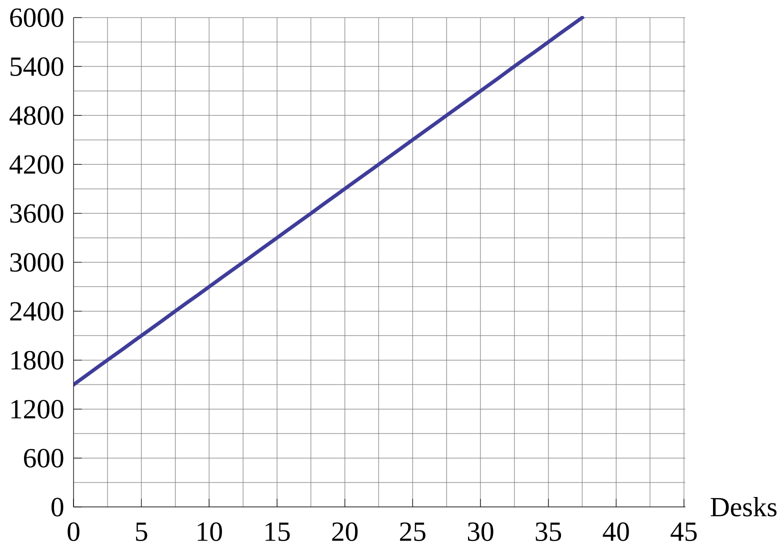
Complete Solution:
The price-demand equation is $p = 2800 - 25x$. If demand $x = 400$, then $p = 2800 - 25(40) = 1800$.

Summary: AmeriCam should charge \$1800 per camera.

18.2 Seduto Inc. makes combination desk-chairs for classrooms. The weekly revenue and cost functions are:

$$\begin{aligned} R(x) &= 480x - 12x^2 \\ C(x) &= 1500 + 120x, \end{aligned}$$

where revenue and cost are measured in dollars and x is the number of desk-chairs produced and sold. The graph of the cost function is shown below:



(a) The graph of the revenue function $R(x)$ is a parabola. Graph the parabola on the same graph.

(b) Label the x-intercepts and vertex with their coordinates.

(c) Draw a bold line to represent the maximum **profit** graphically.

18.3 `[On Webwork]` Sum The cost and revenue functions (in dollars) for producing and selling x Kudsu Sushi machines are given by:

$$C(x) = 40 + 5x, \qquad R(x) = -x^2 + 105x.$$

Find the production level that maximizes profit. Explain your work and give a summary of the cost, revenue, and profit attained at the production level that maximizes profit.

18.4 `[On Webwork]` Sum The cost and revenue functions (in dollars) for producing and selling x Ratmeister hamster cages are given by:

$$C(x) = 600 + 4x, \qquad R(x) = -x^2 + 64x.$$

Find the production level that maximizes profit. Explain your work and give a summary of the cost, revenue, and profit attained at the production level that maximizes profit.

18.5 Expro Inc. manufactures electronic whiteboards. The cost in dollars to produce x whiteboards is given by

$$C(x) = 300 + 4x.$$

and the revenue function is

$$R(x) = 20x - 0.1x^2.$$

(a) Find the profit function $P(x)$.
Answer only:
$P(x) = -0.1x^2 + 16x - 300$

(b) Sum How many whiteboards should be manufactured and sold to maximize profit?
Answer only:
$x = 80$.
Summary: Expro should sell 80 whiteboards to maximize profits

(c) If the government imposes a tax of \$1.00 per whiteboard, this will effect our production costs and thus our profits. As compared to the situation of the old costs, the output level that would maximize profits using the new costs would be

i. higher.
ii. **lower**.
iii. remain the same.

19 Section 4.1 - 4.3: Systems of linear equations, Augmented Mx, Gauss-Jordan

19.1 Solve this system of linear equations:

$$\begin{aligned} 4x + 3y &= 37 \\ -3x + 2y &= -32. \end{aligned}$$

19.2 Solve this system of linear equations:

$$\begin{aligned} 4x + 3y &= -30 \\ -8x - 6y &= 0. \end{aligned}$$

19.3 Find the coordinates (x, y) of the point of intersection for the lines with the equations:

$$\begin{aligned} x + 2y &= -4 \\ 3x + 4y &= -2 \end{aligned}$$

19.4 Find the coordinates (x, y) of the point of intersection for the lines with the equations:

$$\begin{aligned} x + 2y &= 7 \\ 3x + 6y &= 21 \end{aligned}$$

19.5 The supply and demand equations for a product are given below:

$$\begin{array}{lrcr} \text{supply} & q - 3p & = & -1 \\ \text{demand} & q + 2p & = & 4. \end{array}$$

(a) Find an augmented matrix that corresponds to this system of equations.
Complete Solution:
The augmented matrix for the system is

$$A = \begin{bmatrix} 1 & -3 & -1 \\ 1 & 2 & 4 \end{bmatrix}$$

(b) Put the matrix from part (a) in row reduced echelon form.
Answer only:
The row reduced echelon form is

$$R = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}.$$

(Student should show reduction steps.)

(c) How many solutions does the system have?
Complete Solution:
There is only one solution.

19.6 The supply and demand equations for a product are given below:

$$\begin{array}{lrcr} \text{supply} & q - 3p & = & -5 \\ \text{demand} & 5q + 2p & = & 60. \end{array}$$

(a) Find an augmented matrix that corresponds to this system of equations.

(b) Put the matrix from part (a) in row reduced echelon form.

(c) How many solutions does the system have?

19.7 The supply and demand equations for a product are given below:

$$\begin{array}{lrcr} \text{supply} & 10q - 25p & = & -50 \\ \text{demand} & q + p & = & 30. \end{array}$$

(a) Find an augmented matrix that corresponds to this system of equations.
Complete Solution:

$$A = \begin{bmatrix} 10 & -25 & -50 \\ 1 & 1 & 30 \end{bmatrix}$$

(b) Put the matrix from part (a) in row reduced echelon form.
Complete Solution:

$$\begin{bmatrix} 10 & -25 & -50 \\ 1 & 1 & 30 \end{bmatrix} \qquad R_1 \to \frac{1}{5}R_1$$

$$\begin{bmatrix} 2 & -5 & -10 \\ 1 & 1 & 30 \end{bmatrix} \qquad R_2 \to 2R_2 - R_1$$

$$\begin{bmatrix} 2 & -5 & -10 \\ 0 & 7 & 70 \end{bmatrix} \qquad R_2 \to \frac{1}{7}R_2$$

$$\begin{bmatrix} 2 & -5 & -10 \\ 0 & 1 & 10 \end{bmatrix} \qquad R_1 \to R_1 + 5R_2$$

$$\begin{bmatrix} 2 & 0 & 40 \\ 0 & 1 & 10 \end{bmatrix} \qquad R_1 \to \frac{1}{2}R_1$$

$$\begin{bmatrix} 1 & 0 & 20 \\ 0 & 1 & 10 \end{bmatrix} \qquad \text{Row-reduced form}$$

(c) How many solutions does the system have? **Complete Solution:**
The system has one unique solution.

20 Section 4.4: Matrix Operations

20.1 Both of the Mathematics Departments at CSU Northridge and Fullerton give final exams in College Algebra (CA) and the Mathematical Methods for Business (BM). This uses resources from the department faculty (F) to make the exams, the staff (S) to copy the exams and the teaching assistants (T) to proctor the exams. Here are the labor-hour and wage requirements for administering each exam:

	Faculty	Staff	Teaching Assistants
Business Math Exam	5.0 hrs work	0.5 hrs work	2.0 hrs work
College Algebra Exam	7.0 hrs work	1.0 hrs work	2.0 hrs work

	CSUN	CSU, Fullerton
Faculty	\$40 per hour	\$50 per hour
Staff	\$14 per hour	\$16 per hour
Teaching Assistants	\$8 per hour	\$10 per hour

The labor-hours and wage information is given in the following matrices:

$$M = \begin{bmatrix} 5.0 & 0.5 & 2.0 \\ 7.0 & 1.0 & 2.0 \end{bmatrix}, \qquad N = \begin{bmatrix} 40 & 50 \\ 14 & 16 \\ 8 & 10 \end{bmatrix}$$

(a) Compute the product MN
Answer only:

$$MN = \begin{bmatrix} 223 & 278 \\ 310 & 386 \end{bmatrix}$$

(b) What is the $(1, 2)$-entry (also known as R1C2) of matrix MN and what does it mean?
Summary: The $(1, 2)$-entry of MN is 278. \$278 are spent on labor to make up the Business Math Exam at CSU Fullerton.

(c) Delta Duplex Properties builds two-family dwellings. They have two models: Economy Model, Deluxe Model. The cost to build depends on the square footage of the building and the size of the lot. Of course, the Deluxe Model building and lot are larger than the Economy Model. Square footage and costs per square foot are given in the tables below:

	Size of building	Size of lot
Economy Model	2300	7000
Deluxe Model	3000	9000

Sizes are given in square feet.

Building cost	Lot cost
\$300	\$100

Costs are given in dollars per square foot.
The size and cost information is given in the following matrices:

$$S = \begin{bmatrix} 2300 & 7000 \\ 3000 & 9000 \end{bmatrix}, \qquad C = \begin{bmatrix} 300 \\ 100 \end{bmatrix}.$$

(a) Compute the product SC.
(b) Sum Explain what each of the entries in the product SC means.

20.2 There are two food stores near Mrs. Garcia's house, Hons (H) and Trader Vo (TV). She needs to buy 8 apples, 5 bananas, and 2 bunches of cilantro. The prices for each of these items in the two stores are given in the table below:

	Apples	Bananas	Cilantro
H	\$0.40 each	\$0.50 each	\$1.20 each
TV	\$0.55 each	\$0.35 each	\$1.00 each

Mrs. Garcia's shopping list is given in the next table:

Apples	Bananas	Cilantro
8	5	2

The prices and shopping list are given in the following matrices:

$$P = \begin{bmatrix} 0.40 & 0.50 & 1.20 \\ 0.55 & 0.35 & 1.00 \end{bmatrix}, \qquad S = \begin{bmatrix} 8 \\ 5 \\ 2 \end{bmatrix}$$

(a) Compute the product PS
Answer only:

$$\begin{bmatrix} 8.10 \\ 8.15 \end{bmatrix}$$

(b) What is the $(2,1)$-entry (also known as R2C1) of matrix PS and what does it mean?

20.3 CarCoCo (CCC) and AceAuto (AA) are competing auto body shops that specialize in painting cars. Three types of labor are required to complete a paint job: Sanding/Filling, Masking, and Spraying. The number of hours required to complete each job at the two shops are given in the first table and the matrix L. Labor costs, in dollars per hour, are given in the second table and the matrix C.

	Sanding/ Filling	Masking	Spraying
CCC	6	8	2
AA	5	4	2

Sanding/Filling	\$16.00
Masking	\$10.00
Spraying	\$25.00

$$L = \begin{bmatrix} 6 & 8 & 2 \\ 5 & 4 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 16.00 \\ 10.00 \\ 25.00 \end{bmatrix}.$$

A) Compute the matrix product LC

B) What is the (2,1) entry of LC and what does it mean?

20.4 (a) Put the matrix A in row reduced echelon form. Describe each row operation that is used.

$$A = \begin{bmatrix} 1 & 3 & 0 & 5 \\ 4 & 12 & 2 & 0 \end{bmatrix}$$

Answer only:

$$\begin{bmatrix} 1 & 3 & 0 & 5 \\ 0 & 0 & 1 & -10 \end{bmatrix}$$

(b) Compute the following:

$$\begin{bmatrix} 1 & 3 & 2 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 2 \\ 4 & 0 \end{bmatrix} - 3 \begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}$$

Answer only:

$$\begin{bmatrix} 9 & -1 \\ 18 & 2 \end{bmatrix}$$