WARM UP EXERCISE

The ozone level (in parts per billion) on a summer day at R University is given by

$$P(x) = 80 + 12t - t^2$$

Where t is hours after 9 am.

1. P ’ (x).
2. Find P (3) and P ’ (3).
3. Write an interpretation.

Given y = f (x) then the derivative may be represented by any of the following: f ’ (x), y’, $\frac{dy}{dx}$

§10.5 Derivatives of Constants, Power Forms, and Sums

The student will learn about:

- **the derivative of a constant function**
- **the power rule**
- **a constant times f (x)**
- **derivatives of sums and differences**
- **applications**

The Derivative of a Constant

Let f(x) = 5.

What is the slope of a the graph of the constant function?

Theorem 1. Let $y = f(x) = c$ be a constant function, then

$$y' = f'(x) = 0.$$

Power Rule

Example: $f(x) = x^2$

$f'(x) =$

$f(x) = x^3$

$f'(x) = 3x^2$

$f(x) = x^4$

$f'(x) = 4x^3$

$f(x) = x^{1/2}$

$f'(x) = (1/2)x^{-1/2}$

Theorem: If $y = f(x) = x^n$ then $y' = f'(x) = n\,x^{n-1}$.

This works for any real number n (not just integers)

Example 3

f (x) = $\sqrt[3]{x}$
f ' (x) =

f (x) = $x^{2/3}$
f ' (x) =

f (x) = $x^{3/2}$
f ' (x) =

Example 3

$f(x) = 3x^2$

$f'(x) =$

Theorem 3. Let $y = f(x) = k \cdot u(x)$ be a constant k times a differential function u (x). Then

$$y' = f'(x) = k \cdot u'(x) = k \cdot u'.$$

Example 4

$f(x) = 6x^{2/3}$

$f'(x) =$

$f(x) = 4x^{5/3}$

$f'(x) =$

Sum and Difference Properties.

Theorem: If $f(x) = u(x) + v(x)$,

then $f'(x) = u'(x) + v'(x)$.

Example 5

$f(x) = 3x^5 + x^4 - 2x^3 + 5x^2 - 7x + 4$

$f'(x) =$

Applications.

Remember that the derivative gives the instantaneous rate of change of the function with respect to x. That might be:

- Instantaneous velocity
- Tangent line slope at a point on the curve of the function.
- Marginal Cost.
  - If C (x) is the cost function
  - C ’ (x) approximates the cost of producing one more item at a production level of x items.
  - C ’ (x) is called the marginal cost.

Application Example

This example shows the essence in how the derivative is used in business.

The total cost (in dollars) of producing x portable radios per day is

$C(x) = 1000 + 100x - 0.5x^2$ for $0 \le x \le 100$.

1. Find the marginal cost at a production level of x radios.

2. Find the marginal cost at a production level of 80 radios and interpret the result.

Application Example, continued

The total cost (in dollars) of producing x portable radios per day is

$C(x) = 1000 + 100x - 0.5x^2$ for $0 \le x \le 100$.

$C'(x) = 100 - x$, $C'(80) = \$20$

3. Find the actual cost of producing the 81st radio and compare this cost with the previous results.

Summary.

If $f(x) = C$ then $f'(x) = 0$.

If $f(x) = x^n$ then $f'(x) = n\,x^{n-1}$.

If $f(x) = k \bullet u(x)$ then $f'(x) = k \bullet u'(x) = k \bullet u'$.

If $f(x) = u(x) \pm v(x)$, then

$$f'(x) = u'(x) \pm v'(x).$$