§10.4 The Derivative

The student will learn about:

•Rate of change

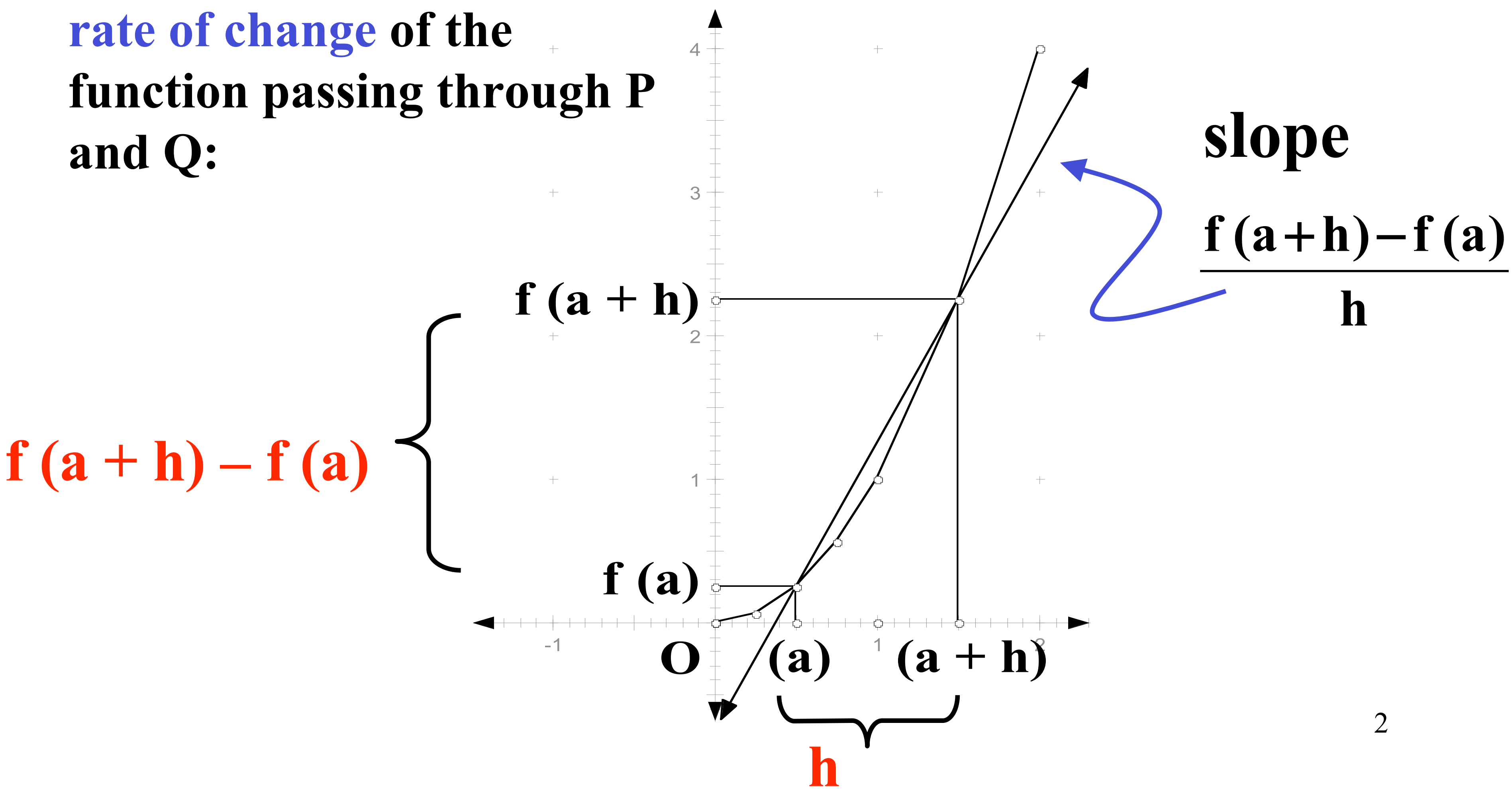
•Slope of the tangent line

•The derivative

•Existence/Nonexistence of the derivative

Difference Quotient: Slope

The difference quotient that follows gives the average rate of change of the function passing through P and Q:



Example 1

The revenue is given by $R(x) = x(75 - 3x)$ for $0 \le x \le 20$.

What is the change in revenue if production changes from 9 to 12?

What is the average rate of change in revenue if production changes from 9 to 12?

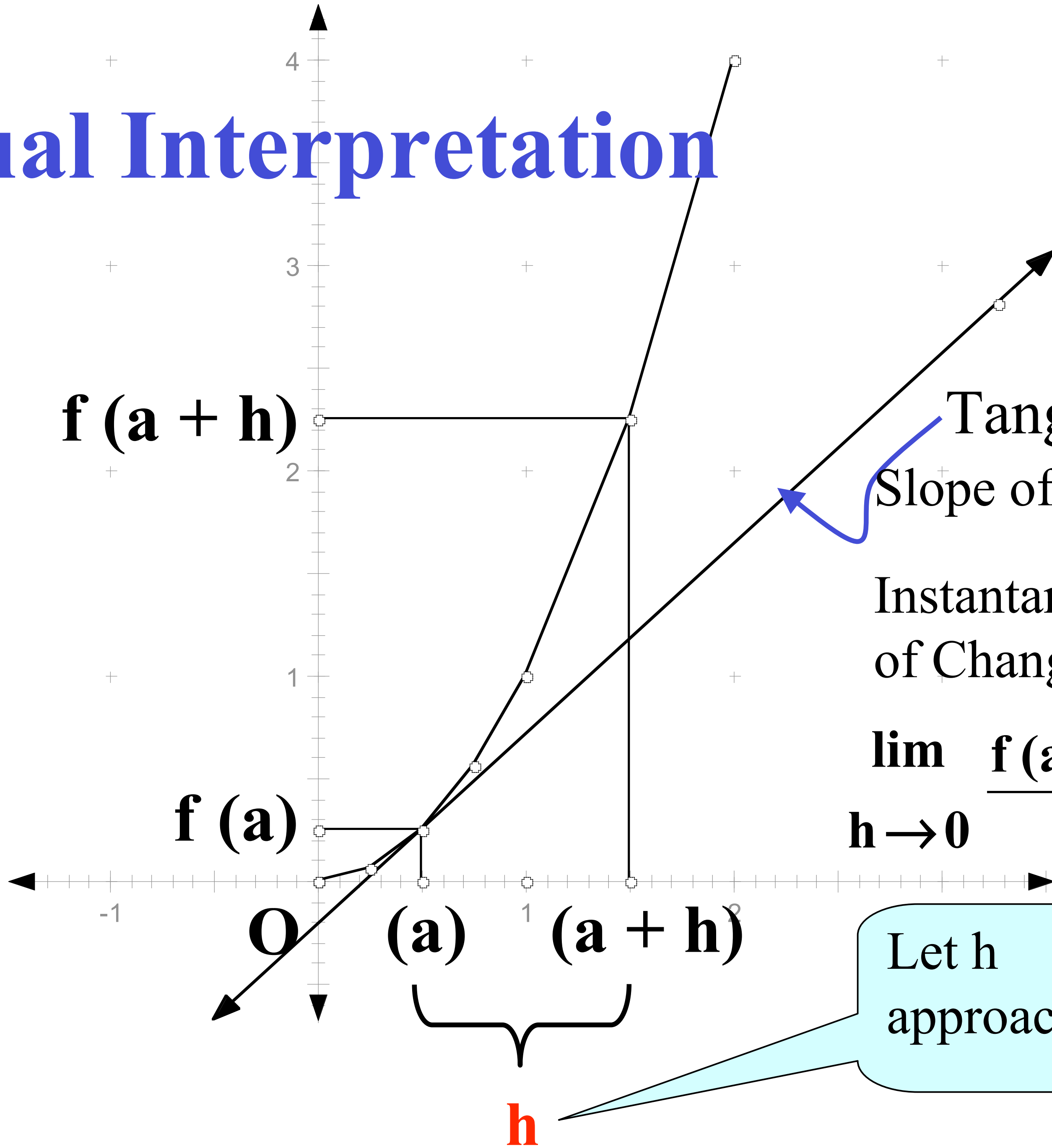
The Instantaneous Rate of Change.

•**Consider the function** ***y = f (x)*** **only at the point** ***(a, f (a))*** **.**

•**The limit of the difference quotient that follows gives the instantaneous rate of change of the function passing through** ***(a, f (a))*****:**

The instantaneous rate of change = $$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Visual Interpretation



Tangent

Slope of tangent.

Instantaneous Rate of Change.

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Let h approach 0

Definition of the Derivative

Given $y = f(x)$, the slope of the graph at the point $(a, f(a))$ is given by

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Provided that the limit exist.

•We define it to be the derivative of *f* at *x*, denote it by $f'(x)$.

•We define the tangent line to $y=f(x)$ at the point $(a, f(a))$ to be the line through this point of slope equal to $f'(x)$.

•$f'(x)$ is the instantaneous rate of change at x (e.g. velocity).

•If $f'(x)$ exists for each x in the open interval (a, b), then f is said to be differentiable over (a, b).

Four-Step Process

To find f ‘ (x) we use a four-step process

Step 1. Find f (x = h)

Step 2. Find f (x + h) – f (x)

Step 3. Find $$\frac{f(x+h)-f(x)}{h}$$

Step 4. Find $$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

Example 2

Find the derivative of $f(x) = x^2 - 3x$

$f(x + h) = (x + h)^2 - 3(x + h)$

$f(x + h) - f(x)$

$$\frac{\mathbf{f(x + h) - f(x)}}{\mathbf{h}}$$

Example 3

Find the slope of the graph of $f(x) = x^2 – 3x$ *at* $x = 0$, $x = 2$, *and* $x = 3$.

From example 2 we found the derivative of this function at x to be
$f'(x) = 2x - 3$

Example 4

$$R(x) = 60x - .02x^2$$

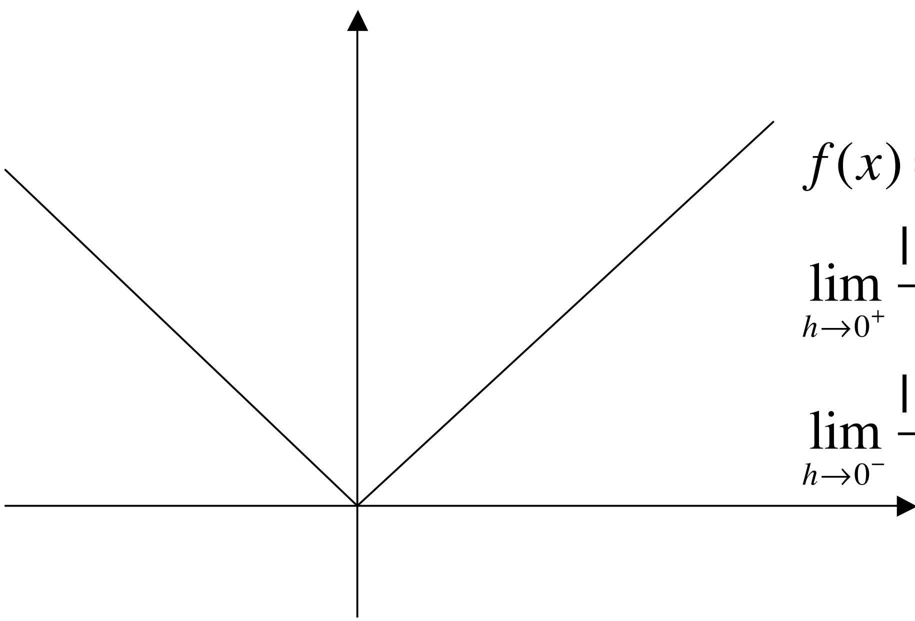
Find R' (x)

Nonexistence of the Derivative.

The existence of a derivative at $x = a$ depends on the existence of a limit at $x = a$; that is, on the existence of

$$\mathbf{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

If the limit does not exist at x = a, we say that the function is nondifferentiable at x = a, or f ' (x) does not exist.



$$f(x) = |x|$$

$$\lim_{h\to 0^+} \frac{|x+h| - |x|}{h} = \text{slope of "right" tangent line=}$$

$$\lim_{h\to 0^-} \frac{|x+h| - |x|}{h} = \text{slope of "left" tangent line=}$$

Nonexistence of the Derivative.

In essence, a derivative of a function does not exist at x = a, if:

- The graph of f has a hole or break at x = a, or if
- The graph of f has a sharp corner at x = a, or if
- The graph of f has a vertical tangent at x = a.

Application

The revenue in dollars from the sale of x car seats for infants is given by $R(x) = 60x - .02x^2$. Find the revenue and the instantaneous rate of change of revenue at a production level of 1000 car seats. What does this mean?